

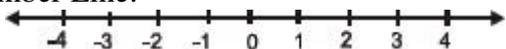
THE SCHRAM ACADEMY

CLASS : IX

NUMBER SYSTEM

MATHEMATICS

Basic Concept and Important Points

- Natural Numbers: Numbers from 1 (one) onward are known as Natural numbers, denoted by 'N'.
 $N = \{1, 2, 3, 4, \dots\}$
- Whole Numbers: Numbers from 0 (zero) onward are known as Whole numbers, denoted by 'W'.
 $W = \{0, 1, 2, 3, 4, \dots\}$
- Integers: The collection of all whole numbers and negative of natural numbers are called Integers, denoted by 'Z' or 'I'.
 $Z \text{ or } I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- Rational Number: A number which can be expressed as $\frac{p}{q}$ where $q \neq 0$ and $p, q \in Z$ is known as rational number, denoted by 'Q'.
- Irrational Number: A number which can't be expressed in the form of p/q and its decimal representation is non-terminating and non-repeating is known as irrational number.
e.g., $\sqrt{2}, \sqrt{3}, \pi, \dots, 1.732105, \text{ etc.}$
- Number Line:

- Method to find two or more rational numbers between two numbers p and q:
If $p < q$, then one of the number be $p < \frac{p+q}{2} < q$ and other will be in continuation as
$$p < \frac{p+(p+q)/2}{2} < \frac{p+q}{2}$$
- The sum of a rational number and an irrational number is always an irrational number.
- The product of a non-zero rational number and an irrational number is always an irrational number.
e.g., $\frac{2}{3}\sqrt{5}$ is an irrational number.
- The sum of two irrational numbers is not always an irrational number.
 $(2+\sqrt{2})+(2-\sqrt{2})=4$ (a rational number)
- The product of two irrational numbers is not always an irrational number.
e.g., $(\sqrt{3}-1)(\sqrt{3}+1)=3-1=2$
- If a is a rational number and n is a positive integer such that the nth root of a is an irrational number, then $a^{1/n}$ is called a surd.
e.g., $\sqrt{5}, \sqrt{2}, \sqrt{3}, \text{ etc.}$
- If $\sqrt[n]{a}$ a is a surd then 'n' is known as order of surd and 'a' is known as radicand.
- Every surd is an irrational number but every irrational number is not a surd.
- Laws of radicals:
(a) $(\sqrt[n]{a})^n = a$ (b) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$
[one of either a or b should be non-negative integer]
(c) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
(d) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$
(e) $\frac{\sqrt[p]{a^n}}{\sqrt[p]{a^m}} = \sqrt[p]{a^{n-m}}$
(f) $\sqrt[p]{a^n \times a^m} = \sqrt[p]{a^{n+m}}$
(g) $\sqrt[p]{(a^n)^m} = \sqrt[p]{a^{n.m}}$

- A surd which has unity only as rational factor is called a pure surd.
- A surd which has a rational factor other than unity is called a mixed surd.
- Order of a given surd can be changed by using following steps:
 - (a) Let the surd be $\sqrt[n]{a}$ and m be the order of surd to which it has to be converted.
 - (b) Compute $\frac{m}{n}$ and let $\frac{m}{n} = p$.
 - (c) Write $\sqrt[n]{a} = \sqrt[m]{a^p}$ which is the required result.
- Surds having same irrational factors are called similar or like surds.
- Only similar surds can be added or subtracted by adding or subtracting their rational parts.
- Surds of same order can be multiplied or divided.
- If the surds to be multiplied or to be divided are not of the same order, we first convert them to the same order and then multiply or divide.
- If the product of two surds is a rational number, then each one of them is called the rationalising factor of the other.

e.g., $\sqrt[3]{2} \times \sqrt[3]{4} = 2$, then $\sqrt[3]{2}$ and $\sqrt[3]{4}$ are rationalising factors of one another.
- A surd consisting of one term only is called a monomial surd.
- An expression consisting of the sum or difference of two monomial surds or the sum or difference of a monomial surd and a rational number is called binomial surd.

e.g.,

$$\sqrt{2} + \sqrt{5}, \sqrt{3} + 2, \sqrt{2} - \sqrt{3},$$

etc., are binomial surds.
- the binomial surds which differ only in sign (+ or -) between the terms connecting them, are called conjugate surd

e.g.,

$$\sqrt{3} + \sqrt{2} \text{ and } \sqrt{3} - \sqrt{2}$$

or $2 + \sqrt{5}$ and $2 - \sqrt{5}$ are conjugate surds.
- Rational exponents:
 - (a) If x, y be any rational numbers different from zero and m be any integer, then $x^m \times y^m = (x \times y)^m$.
 - (b) If x be any rational number different from zero and m, n be any integers, then $x^m \times x^n = x^{m+n}$ and $(X^m)^n = x^{m \times n}$.
- Reciprocals of positive integers as exponents:

If q be any positive integer other than 1, and x and y be rational numbers such that $x^q = y$ then $y^{1/q} = x$. We write $y^{1/q}$ as $y^{1/q}$ as $\sqrt[q]{y}$ and read it as q th root of y . $\sqrt[q]{y}$ is called a radical and q is called the index of the radical.
- Positive rational numbers as exponents:

If $\frac{p}{q}$ be any positive rational number (where p and q are positive integers prime to each other) and let x be any rational number. We have already given a meaning to $x^{p/q}$. This can be done very easily.

That is $x^{p/q}$ is the q th root of x^p .

Thus, $(4)^{3/2} = (4^3)^{1/2} = (64)^{1/2} = 8$.
- If $\frac{p}{q}$ is a negative rational number, then $x^{p/q}$ ($x \neq 0$) is equal to $\frac{1}{x^{-p/q}}$.
- If x be any rational number different from zero, and a and b be any rational numbers, then $x^a \div x^b = x^{a-b}$.

