THE SCHRAM ACADEMY

FORMULAE SHEET – CLASS 12

INSTRUCTIONS TO BE FOLLOWED

Students should learn and practice the formulae without looking at the formulae sheet for the chapters

1. VECTOR ALGEBRA and THREE DIMENSIONAL GEOMETRY by 21/03/2020
2. PROBABILITY, TRIGONOMETRY and INVERSE TRIGONOMETRIC FUNCTIONS by 24/03/2020
3. MATRICES, DETERMINANTS and RELATIONS & FUNCTIONS by 27/03/2020
4. DIFFERENTIATION and INTEGRATION by 31/03/2020

FORMULAE – CLASS 12

MATRICES

1. A+B = B+A
2. (A+B)+C = A+(B+C)
3. 0 is the additive identity
4. –A I s the additive inverse of A.
5. A_{mxn}, B_{nxl} then AB_{mxl}
6. (AB)C=A(BC)
7. A(B+C)=AB+AC
8. IA=AI=A
9. (A')' =A
10. (KA)'=KA' where K is a constant.
11. (A+B)'=A'+B'
12. (AB)'=B'A'
13. If A'=A then A is symmetric
14. If A'= - A then A is skew symmetric
15. A+A' is symmetric
16. A - A' is skew symmetric
17. A= $\frac{1}{2}$ (A+A') + $\frac{1}{2}$ (A-A')
18. If AB = BA = I then A is the inverse of B (or) B is inverse of A
19. $(AB)^{-1}=B^{-1}A^{-1}$
20. $|kA| = k^n|A|$, where n is the order of matrix A

INVERSE TRIGONOMETRIC FUNCTIONS

1. $\sin^{-1}(\frac{1}{x}) = \operatorname{cosec}^{-1}x$, $-1\leq x\leq 1$
2. $\cos^{-1}(\frac{1}{x}) = \sec^{-1}x$, $-1\leq x\leq 1$
3. $\tan^{-1}(\frac{1}{x}) = \cot^{-1}x$, $x > 0$
4. $\sin^{-1}(-x) = -\sin^{-1}x$, $x\in[-1,1]$
5. $\tan^{-1}(-x) = -\tan^{-1}x$, $x\in R$
6. $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$, $|x| \geq 1$
7. $\cos^{-1}(-x) =\pi-\cos^{-1}x$, $x\in[1,1]$
8. $\sec^{-1}(-x) = \pi-\sec^{-1}x$, $|x| \geq 1$
9. $\cot^{-1}(-x) = \pi-\cot^{-1}x$, $x\in R$
10. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $x\in[-1,1]$
11. $\tan^{-1}x +\cot^{-1}x = \frac{\pi}{2}$, $x\in R$
12. $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$,$|x| \geq 1$
13. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, $xy <1$
14. $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, $xy> -1$
15. $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $|x| \leq 1$
16. $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $x\geq 0$
17. $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, $-1<x< 1$

TRIGONOMETRY

1. sin(A+B) = sin A cos B +cos A sin B
2. sin(A-B) = sin A cos B – cos A sin B
3. cos(A+B) = cos A cos B – sin A sin B
4. cos(A-B) = cos A cos B + sin A sin B
5. $\sin C+ \sin D = 2 \sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
6. $\sin C- \sin D = 2 \cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
7. $\cos C+ \cos D = 2 \cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
8. $\cos C- \cos D = -2 \sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
9. sin2A = 2sinAcosA (or) $\frac{2tanA}{1+tan^2A}$
10. $\cos 2A = \cos^2A - \sin^2A$

    $= 2\cos^2A-1$ (or) $1-2\sin^2A$ (or) $\frac{1-tan^2A}{1+tan^2A}$

11. $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

12. $2\sin^2 A = 1-\cos 2A$ & $2\sin^2(\frac{A}{2}) = 1-\cos A \Rightarrow \sin^2(\frac{A}{2}) = \frac{1-\cos A}{2}$

13. $2\cos^2 A = 1+\cos 2A$ & $2\cos^2(\frac{A}{2}) = 1+\cos A \Rightarrow \cos^2(\frac{A}{2}) = \frac{1+\cos A}{2}$

14. $\sin 3A = 3\sin A - 4\sin^3 A$

$\sin^3 A = \frac{(3\sin A - \sin 3A)}{4}$

15. $\cos 3A = 4\cos^3 A - 3\cos A$

$\cos^3 A = \frac{\cos 3A + 3\cos A}{4}$

16. $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

17. $\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

18. $\cos(A+B) - \cos(A-B) = -2\sin A\sin B$

19. $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

20. $\sin(A+B) - \sin(A-B) = 2\cos A\sin B$

21. $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$

22. $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$

23. $\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}$

24. $\cot(A-B) = \frac{\cot A\cot B + 1}{\cot A - \cot B}$

Note :- $\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

25. $1+\sin 2x = (\sin x + \cos x)^2$

26. $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

27. $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

28. $\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$

29. $\tan(\frac{\pi}{4}+x) = \frac{1+\tan x}{1-\tan x}$

30. $\tan(\frac{\pi}{4}-x) = \frac{1-\tan x}{1+\tan x}$

SOLVING TRIGONOMETRIC EQUATIONS

1. $\sin x=0$ then $x = n\pi$
2. $\cos x = 0$ then $x = (2n+1)\frac{\pi}{2}$
3. $\tan x = 0$ then $x = n\pi$
4. $\sin x= \sin y$ then $x = n\pi+(-1)^n y$
5. $\cos x = \cos y$ then $x = 2n\pi \pm y$
6. $\tan x= \tan y$ then $x = n\pi + y$

Note:- y is the least principal value, $n\in Z$

TRIGONOMETRIC IDENTITIES

1. $\sin^2 x+\cos^2 x = 1$
2. $\sec^2 x-\tan^2 x = 1$
3. $\operatorname{cosec}^2 x - \cot^2 x = 1$

DETERMINANTS

1. Area of triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Note: Area is a positive quantity but when area is given, take both positive and negative values of the determinants for the calculation

2. If $|A| = 0$, A is called singular matrix

   If $|A| \neq 0$, A is called non - singular matrix
3. $|AB| = |A|\,|B|$
4. $A(\operatorname{adj}A) = (\operatorname{adj}A)A = |A|\,I$
5. $(\operatorname{adj}A) = |A|^{n-1}$
6. $\operatorname{adj}(AB) = (\operatorname{adj}B)(\operatorname{adj}A)$
7. $\operatorname{adj}(A^T) = (\operatorname{adj}A)^T$
8. $\operatorname{adj}(\operatorname{adj}A) = |A|^{n-2}A$
9. $A^{-1} = \frac{1}{|A|}(\operatorname{adj}A)$
10. $(A^{-1})^{-1} = A$

11. $(AB)^{-1} = B^{-1} A^{-1}$
12. $(A^T)^{-1} = (A^{-1})^T$
13. If A is non singular matrix then $|A^{-1}| = \frac{1}{|A|}$
14. Matrix method of solving system of equations is $X = A^{-1}B$, provided $|A| \neq 0$
Note: If $|A| = 0$ then calculate (adjA)B
(i) If (adjA)B$\neq$ 0 then the system of equations is inconsistent
(ii) If (adjA)B = 0 then it has either infinitely many solutions or no solution
1. log (ab) = log a + log b
2. $\log\left(\frac{a}{b}\right) = \log a - \log b$
3. $\log(a^b) = b \log a$
4. $\log_b a = \frac{1}{\log_a b}$

DIFFERENTIATION

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(ax+b)^n = n(ax+b)^{n-1}.a$
3. $\frac{d}{dx}(e^x) = e^x$
4. $\frac{d}{dx}(a^x) = a^x \log_e a$
5. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$
6. $\frac{d}{dx}(\log_b x) = \frac{1}{x \log_e b}$
7. $\frac{d}{dx}(sinx) = \cos x$
8. $\frac{d}{dx}(cosx) = -\sin x$
9. $\frac{d}{dx}(\tan x) = \sec^2 x$
10. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
11. $\frac{d}{dx}(\sec x) = \sec x \tan x$
12. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
13. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$
14. $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$
15. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$
16. $\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$
17. $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$
18. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

LIMITS

1. $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$
2. $\lim_{x\to 0}\frac{sinx}{x} = 1$
3. $\lim_{x\to 0}\frac{tanx}{x} = 1$
4. $\lim_{x\to 0}\frac{e^x-1}{x} = 1$
5. $\lim_{x\to 0}\frac{\log|1+x|}{x} = 1$
6. $\lim_{x\to 0}\frac{a^x-1}{x} = \log_e a$
7. $\lim_{x\to 0}\frac{sin^{-1}x}{x} = 1$
8. $\lim_{x\to 0}\frac{tan^{-1}x}{x} = 1$
9. Left derivative at a point x = c
$\lim_{h\to 0}\frac{f(c-h)-f(c)}{-h}$

10. Right derivative at a point x=c

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

Note : for a function f(x) to be differentiable at a point x=c is left hand derivative = right hand derivative

APPLICATION OF DERIVATIVES

1. Area of square = x^2 , Perimeter of a square = 4x
2. area of rectangle = xy , Perimeter of a rectangle =2(x+y)
3. area of a trapezium = $\frac{1}{2}$ x Sum of parallel sides x Perpendicular distance between them
4. area of circle =πr^2 cirumferences of o le =2 πr
5.

	CSA	TSA	Volume
Cylinder	$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
Cone	πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
Sphere	$4\pi r^2$	-	$\frac{4}{3}\pi r^3$
hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

6. Volume of parallelepiped = xyz , S.A of parallelopiped = 2(xy+yz+zx)
7. Volume of cube = x^3 , S.A of cube =$6x^2$
8. Area of equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2$

INTEGRATION

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1}$ +c
2. $\int dx$ = x+c
3. $\int \sin x\,dx = -\cos x + c$
4. $\int \cos x\,dx = \sin x + c$
5. $\int \sec^2 x\,dx = \tan x + c$
6. $\int \sec x \tan x\,dx = \sec x + c$
7. $\int \mathrm{cosec}^2 x\,dx = -\cot x + c$
8. $\int \mathrm{cosec}\,x \cot x\,dx = -\mathrm{cosec}\,x + c$
9. $\int e^x\,dx = e^x + c$
10. $\int a^x\,dx = \frac{a^x}{\log a} + c$
11. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$
12. $\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1}x + c$
13. $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$
14. $\int \frac{-1}{1+x^2}\,dx = \cot^{-1}x + c$
15. $\int \frac{1}{x\sqrt{1-x^2}}\,dx = \sec^{-1}x + c$
16. $\int \frac{-1}{x\sqrt{1-x^2}}\,dx = \mathrm{cosec}^{-1}x + c$
17. $\int \tan x\,dx = \log|\sec x| + c$ (or) $-\log|\cos x| + c$
18. $\int \log x\,dx$ u=logx $\int dv = \int dx$

du= $\frac{1}{x}$ dx v = x

$\int \log x\,dx = x\log x - \int x.\frac{1}{x}\,dx$

$\therefore \int \log x\,dx = x\log x - x + c$

19. $\int \cot x\,dx = \log|\sin x| + c$
20. $\int \sec x\,dx = \log|\sec x + \tan x| + c$

$=\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + c$

21. $\int \mathrm{cosec}\,x\,dx = \log|\mathrm{cosec}\,x - \cot x| + c$

$= \log\left|\tan\left(\frac{x}{2}\right)\right| + c$

22. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
23. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$
24. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$
25. $\int \frac{dx}{\sqrt{a^2+x^2}} = \log\left|x+\sqrt{a^2+x^2}\right| + c$
26. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

27. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log |x+\sqrt{x^2-a^2}| + c$

28. $\int \sqrt{a^2+x^2}\, dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log|x+\sqrt{a^2+x^2}| + c$

29. $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

30. $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}| + c$

31. $\int e^x[f(x) + f'(x)]\, dx = e^x f(x) + c$

32. Integration as a limit of sum

$\int_a^b f(x)\, dx = \lim_{h\to 0} h\,[f(a) + f(a+h) + f(a+2h) + \ldots f[a+(n-1)h]]$

where $h = \frac{b-a}{n}$

Note: (i) $\sum(n-1) = \frac{n(n-1)}{2}$

(ii) $\sum(n-1)^2 = \frac{n(n-1)(2n-1)}{6}$, (iii) $\sum(n-1)^3 = \left[\frac{n(n-1)}{2}\right]^2$

33. $\int e^{ax}\cos bx\, dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$

34. $\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$

Note : sum to G.P

$a+ar+\ldots\ldots+ar^{n-1} = \frac{a(r^n-1)}{r-1}$

PROPERTIES OF DEFINITE INTEGRRALS

(i) $\int_o^a f(x)\, dx = \int_o^a f(a-x)\, dx$

(ii) $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

(iii) $\int_o^{2a} f(x)\, dx = \int_o^a f(x)\, dx + \int_o^a f(2a-x)\, dx$

(iv) $\int_o^{2a} f(x)\, dx = 2\int_o^a f(x)\, dx$ if $f(2a-x) = f(x)$

(v) $\int_o^{2a} f(x)\, dx = 0$ if $f(2a-x) = -f(x)$

(vi) $\int_{-a}^a f(x)\, dx = 2\int_o^a f(x)\, dx$, if f is an even function

= 0, if f is an odd function

(vii) $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

(viii) $\int_{x=a}^{x=b} f(x)\, dx = \int_{t=a}^{t=b} f(t)\, dt$

VECTOR ALGEBRA

1. If $\vec{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ then a,b,c are called Direction ratios and

$l\,(\cos\alpha) = \frac{a}{\sqrt{a^2+b^2+c^2}}$

$m(\cos\beta) = \frac{b}{\sqrt{a^2+b^2+c^2}}$

$n\,(\cos\gamma) = \frac{c}{\sqrt{a^2+b^2+c^2}}$ are direction cosines.

2. If $\overrightarrow{OA} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$

$\overrightarrow{OB} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$ then Dr's of $\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

3. If $\vec{a} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$ then magnitude of $\vec{a}$ is $|\vec{a}| = \sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}$

4. Unit vector in the direction of $\vec{a}$ is $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

Unit vector ll^el to $\vec{a} = \pm\frac{\vec{a}}{|\vec{a}|}$

5. If $\vec{a}$, $\vec{b}$ are two vectors then $\vec{a}\,.\,\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ and

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$ where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$.

6. Unit vector perpendicular to two vectors $\vec{a}$ & $\vec{b}$ is $\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

7. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|}$

8. Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$= \frac{1}{2}|\overrightarrow{BC}\times\overrightarrow{BA}|$

$= \frac{1}{2}|\overrightarrow{CA}\times\overrightarrow{CB}|$

9. Area of $\|^{gm}$ with $\vec{a}$, $\vec{b}$ as adjacent sides = $|\vec{a}\times\vec{b}|$

10. Area of $\|^{gm}$ with $\vec{d}_1$, $\vec{d}_2$ as diagonals = $\frac{1}{2}|\vec{d_1}\times\vec{d_2}|$

11. Area of a triangle with positive vectors of vertices of A,B,C as $\vec{a}$, $\vec{b}$, $\vec{c}$

is $\frac{1}{2}|(\vec{a}\times\vec{b}) + (\vec{b}\times\vec{c}) + (\vec{c}\times\vec{a})|$

Two vectors $\vec{a}$ and $\vec{b}$ collinear vector then $\vec{a} = \pm\lambda\vec{b}$

12. Angle between two vectors $\vec{a}$, $\vec{b}$

$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$

$\theta = \sin^{-1}\left(\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\right)$ (This will give only acute angles).

PROPERTIES

1. If two vectors $\vec{a}$ & $\vec{b}$ are perpendicular then $\vec{a}\cdot\vec{b} = 0$
2. If two vectors $\vec{a}$ & $\vec{b}$ are parallel or collinear, $\vec{a} \times \vec{b} = \vec{0}$
3. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

   $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$
4. $\vec{a}\cdot\vec{a} = |\vec{a}|^2 = a^2$

   $\vec{a} \times \vec{a} = \vec{o}$
5. $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$

   $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$

   $(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2 = a^2 - b^2$

   $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

SCALAR TRIPLE PRODUCT

13. $(\vec{a}\times\vec{b})\cdot\vec{c}$ (or) $\vec{a}\cdot(\vec{b}\times\vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}]$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

14. If $[\vec{a}\ \vec{b}\ \vec{c}] = 0$ then $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar

15. $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$

16. Volume of parallelopiped with $\vec{a}$, $\vec{b}$, $\vec{c}$ are three adjacent sides

    $= |[\vec{a}\ \vec{b}\ \vec{c}]|$

17. Direction ratios of line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is

    $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$

18. Vector equation of a line is $\vec{r} = \vec{a} + \lambda\vec{b}$ where $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ is the position vector of the point through which the line passes and $\vec{b}$ is a ll^{el} vector to the line

19. <u>Cartesian equation of a line is</u>

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

where (x_1, y_1, z_1) is the point through which the line passes and (a, b, c) are dr's of the ll^{el} vector to the line.

20. Angle between two lines with D.r's (a_1, b_1, c_1) and (a_2, b_2, c_2) is

$$\theta = \cos^{-1}\left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)$$

(i) If lines are ⊥r, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(ii) If lines are llel , $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

21. Vector equation of line passing through 2 points with P.V'S $\vec{a}$, $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

22. Cartesian equation of line passing through 2 points (x_1,y_1,z_1) and (x_2,y_2,z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

23. If $\vec{r}_1 = \vec{a}_1 + \lambda\vec{b}_1$

    $\vec{r}_2 = \vec{a}_2 + \lambda\vec{b}_2$ then angle between them is $\theta = \cos^{-1}\left(\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right)$

24. Shortest distance between skew lines (vector form)

    $r_1 = \vec{a}_1 + \lambda\vec{b}_1$, $\vec{r}_2 = \vec{a}_2 + \mu\vec{b}_2$ is $\left|\frac{(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right|$

    Note: If two lines are intersecting then $(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2}) = 0$

25. Shortest distance between ll^{el} lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$, $\vec{r} = \vec{a_2} + \mu\vec{b}$

    $= \left|\frac{(\vec{a_2}-\vec{a_1})\times\vec{b}}{|\vec{b}|}\right|$

26. Shortest distance between skew lines (Cartesian form)

$$\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right|$$

27. Equation of plane passing through $\vec{a}$ and ⊥r to $\vec{n}$ is

    $\vec{r}\cdot\vec{n} = \vec{a}\cdot\vec{n}$ (vector form)

    $A(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ (Cartesian form)

28. Eqn of plane in normal form

    <u>Vector from:-</u> $\vec{r}\cdot\hat{n} = d$ where $\hat{n}$ is a unit normal from origin, d is the⊥r distance from the origin.

    <u>Cartesian form</u>:- $lx + my + nz = d$ where l,m,n are D.c's of $\hat{n}$

29. Equation of plane passing through three points

Vector from:- $(\vec{r}-\vec{a}).\{(\vec{b}-\vec{a}) \times (\vec{c}-\vec{a})\}=0$

Cartesian form:- $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$

30. Condition for two lines $\vec{r}_1 = \vec{a}_1 + \lambda \vec{b}_2$ to be coplanar

Vector from:- $(\vec{a_2}-\vec{a_1}).(\vec{b_1} \times \vec{b_2})=0$

Cartesian form:- $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

31. Plane passing through the intersection of two planes :- $\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is :- Vector from:- $\vec{r}.(\vec{n_1}+\lambda\vec{n_2}) = d_1+\lambda d_2$

Cartesian form:- $(A_1x + B_1y + C_1z - d_1) + \lambda (A_2x + B_2y + C_2z - d_2)=0$

32. Angle between two planes :-

Vector from:- $\cos\theta = \left|\frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|}\right|$

Note :- If $\vec{n_1}.\vec{n_2}=0$ then planes are ⊥r. If $\vec{n_1} = \lambda\vec{n_2}$ then planes are ll[el]

Cartesian form:- $\cos\theta = \left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

Note :- If $A_1A_2 + B_1B_2 + C_1C_2 = 0$ then planes are ⊥r.

If $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$ then planes are ll[el].

33. Distance of a point from plane $\vec{r}.\vec{N} = d$

Vector from:- $\left|\frac{\vec{a}.\vec{N}-d}{\vec{N}}\right|$

Cartesian form:- $\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$

Note :- If $\vec{a}$ is origin then

Vector from:- $\frac{|\vec{d}|}{|\vec{N}|}$

Cartesian form:- $\left|\frac{D}{\sqrt{A^2+B^2+C^2}}\right|$

34. Angle between a line and a plane $(\vec{r} = \vec{a} + \lambda\vec{b})$ & $(\vec{r}.\vec{n}=d)$

$\sin\phi = \left|\frac{\vec{b}.\vec{n}}{|\vec{b}|\,|\vec{n}|}\right|$

PROBABILITY

Conditional Probability

1. $P(A/B) = \frac{P(A\cap B)}{P(B)}$, $P(B) \neq 0$

$P(B/A) = \frac{P(A\cap B)}{P(A)}$, $P(A) \neq 0$

2. Multiplication Theorem

$P(A\cap B) = P(A/B) . P(B)$

(or) $P(B/A) . P(A)$

$P(A\cap B\cap C)=P(A). P(A/B) . P(C/A\cap B)$

Note: (i) If A and B are Independent then $P(A\cap B) = P(A).P(B)$

(ii) If A and B are then $P(A\cup B) = 1- P(A') P(B')$

(iii) If A and B are mutually exclusive then $P(A\cup B) = P(A)+ P(B)$

3. If A and B are independent then A and B', B' and A , A'and B' are also independent.

4. Demorgan's law:- $(A\cup B)' = A'\cap B'$, $(A\cap B)' = A'\cup B'$

Total probability

Let E_1, E_2, E_3, ….. , E_n form a partition of the sample space S of an experiment. If A is any event associated with sample space S, then

$P(A) = P(A/E_1).P(E_1) + P(A/E_2).P(E_2) + \ldots\ldots + P(A/E_n).P(E_n)$

Bayes theorem :-

$P(E_i/A) = \frac{P(A/Ei).P(Ei)}{P(A)}$

Mean of random variable X is $E(X) = \sum x_i p_i$

Variance of a random variable X is

$\sigma^2 = Var(X) = E(X^2) - [E(X)]^2$

Where $E(X^2) = \sum x_i^2 p_i$, **Note:-** S.D = $\sqrt{variance.}$

Binomial distribution

B(n, p)

$P(X = x) = nC_x\ p^x\ q^{n-x}$ where x= 0,1,…..n and p + q =1

Note:- (i) $P(x>a)= 1- p(x\leq a)$

$P(x\geq a)= 1- p(x<a)$

(ii) the word atleast stands for $x\geq a$ and atmost stands for $x \leq a$

ALGEBRAIC IDENTITIES

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2- 2ab+b^2$

$a^2 - b^2 = (a+b)(a-b)$

$(a+b)^3 =a^3+b^3+3ab(a+b)$

$(a-b)^3 =a^3-b^3-3ab(a-b)$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

RELATIONS AND FUNCTIONS

1. Number of relations in a given set A to set B is $2^{n(A) \times n(B)}$
2. If n(A) = p and n(B) = q then
   Number of functions from A to B = q^p
   Number of one to one functions = $\begin{cases} 0 & if\ p > q \\ \frac{q!}{(q-p)!} & if\ p \leq q \end{cases}$
   Number of onto functions = $\begin{cases} \sum_{r=1}^{q} (-1)^{q-r}\ qC_r\ r^p & if\ p \geq q \\ 0 & if\ p < q \end{cases}$
3. Number of binary operations on set containing n elements = n^{n^2}
   Number of commutative binary operations containing n elements = $n^{\frac{n(n-1)}{2}}$
4. If A and B are two finite sets and f : A→B then

(i) f is one to one ⇒ $n(A) \leq n(B)$

(ii) f is on to ⇒ $n(B) \leq n(A)$

(iii) f is bijective ⇒ $n(A) = n(B)$